647510 **BSCbooCC5030** Seat No :________

# B.Sc. Semester - 5 (*CBCS*) Examination
## Oct/Nov. - 2021(Old Course)
## BOOLEAN ALGEBRA AND COMPLEX ANALYSIS-1 (CORE)

**Time: 2:30 Hours** **Marks: 70**

**Instructions:**

1. All questions are compulsory.
2. Figures to the right indicate marks.

---

**1 (A) Answer any one:** **(07)**

(1) If $(L, \leq)$ is a lattice and $a, b, c \in L$ then prove that
(i) $b \leq c \implies a * b \leq a * c$ and $a \oplus b \leq a \oplus c$.
(ii) $a \oplus (b * c) \leq (a \oplus b) * (a \oplus c)$ and
$a * (b \oplus c) \geq (a * b) \oplus (a * c)$

(2) Define: Direct product of two lattice. Prove that direct product of two lattices is also a lattice.

**(B) Answer any one:** **(04)**

(1) Define: Maximal and minimal elements of a poset. Draw the Hasse diagram of $(P(A), \subset)$, where $A = \{1,2,3\}$. Also determine its maximal and minimal elements.

(2) Define: Complemented lattice. Give an example of a bounded lattice which is not a complemented lattice.

**(C) Answer any one:** **(03)**

(1) Define: Chain. Prove or disprove: $(P(A), \subset)$, where $A = \{a, b\}$ is a chain. Justify your answer.

(2) Define: Least element in poset. The least element of poset $(\{1,2,3,4,5,6\}, /) =$ _____

**2 (A) Answer any one:** **(07)**

(1) Define: Boolean algebra. If $(B, *, \oplus, ', 0, 1)$ is Boolean algebra and $a, b \in B$ then prove that:
$$a \leq b \iff a * b' = 0 \iff b' \leq a' \iff a' \oplus b = 1.$$

(2) State and prove unique representation theorem in Boolean algebra

**(B) Answer any one:** **(04)**

(1) Prove that: (i) $(x_1 * x_2 * x_3) \oplus (x_1 * x_2{}' * x_3) = x_1 * x_3$
(ii) $(a * b) \oplus (a * b') \oplus (a' * b) \oplus (a' * b') = 1$

(2) Express the Boolean expression $\alpha(x_1, x_2, x_3) = x_1 x_2$ as the product of sum canonical form.

**(C) Answer any one:** **(03)**

(1) If $(B, *, \oplus, ', 0, 1)$ is Boolean algebra and $x \in B$, define $A(x)$ and show that $A(x') = A - A(x)$.

(2) Write all minterms of Boolean algebra expression consisting three variables.

**3(A) Answer any one.** **(07)**

(1) Define: Laplace Equation and write Cartesian form of Laplace equation and derive Laplace's equation in polar form.

(2) Define: Derivative of complex function at $z_0$. If the complex function $f(z) = u + iv$ is analytic at point $z_0 = x_0 + iy_0$ then prove that $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$ and $\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$.

**(B) Answer any one.** **(04)**

(1) Define: Harmonic Function and find an analytic function whose real part is $cosx\ coshy$.

(2) Find an analytic function $f(z) = u + iv$ such that $Re\big(f'(z)\big) = 3x^2 - 4y - 3y^2$ and $f(1 + i) = 0$.

**(C) Answer any one.** **(03)**

(1) In usual notation prove that $\nabla^2 = 4\frac{\partial^2}{\partial z\, \partial \bar{z}}$.

(2) If $f(z) = x^2 + ayx + by^2 + i(x^2 + cxy + dy^2)$ is analytic then find value of $a, b, c$ & $d$.

**4(A) Answer any one.** **(07)**

(1) State and prove Cauchy's integral formula.

(2) Prove that $\int_c \pi e^{\pi \bar{z}}\, dz = 4(e^{\pi} - 1)$, where c is the square joining points $z = 0,\ z = 1,\ z = 1 + i\ and\ z = i$.

**(B) Answer any one.** **(04)**

(1) In usual notation prove that $\left|\int_a^b f(z)dz\right| \leq \int_a^b |f(z)|\, dz$.

(2) Evaluate: $\int_0^{\pi+2i} \cos\left(\frac{z}{2}\right) dz$.

**(C) Answer any one.** **(03)**

(1) Find $\int_c f(z)dz$, where $f(z) = 4y$; y > 0 and $f(z) = 1$; $y < 0$ and c is a part of curve $y = x^3$ from $-1 - i\ to\ 1 + i$.

(2) Show that: $\left|\int_c \frac{dz}{z^2-1}\right| \leq \frac{\pi}{3}$, where c is the arc from $z = 2\ to\ z = 2i$ of circle $|z| = 2$.

**5(A) Answer any one.** **(07)**

(1) State and prove Cauchy's integral formula for derivatives and deduce that $f^n(z_0) = \frac{n!}{2\pi i}\int_c \frac{f(z)}{(z-z_0)^{n+1}} dz$.

(2) State and prove Morera's theorem.

**(B) Answer any one.** **(04)**

(1) Prove that $\int_c \frac{e^{kz}}{z} dz = 2\pi i, c$: $|z| = 1$ and hence deduce that:

(i) $\int_0^{\pi} e^{kcos\theta} \cos(ksin\theta)\, d\theta = \pi$

(ii) $\int_{-\pi}^{\pi} e^{kcos\theta} \sin(ksin\theta)\, d\theta = 0$

(2) Find: $\int_c \frac{dz}{(z^2+4)^2}$, $C : |z - i| = 2$

**(C) Answer any one.** **(03)**

(1) Find $\int_c \frac{\cosh z}{z^3} dz$, $C : |z| = 1$

(2) Evaluate: $\int_c \frac{e^2 dz}{(z-4)(z-2)}$, $C : |z| = 3$

*******************************